RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER END SEM EXAMINATION, MARCH 2021
FIRST YEAR [BATCH 2020-23]

Date : 30/03/2021 **MATHEMATICS GENERAL**
Time : 11.00 am – 1.00 pm **Paper** : MAGT 1 Full Marks: 50

Instructions to the Candidates

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group: A
Geometry

Answer any **three** questions from question no. 1-5 in this group. [3 x 5 = 15 marks]

1. Reduce the equation $8x^2 - 12xy + 17y^2 + 16x - 12y + 3 = 0$ to its canonical form and state the type of the conic. [5]
2. Show that the equation of the lines passing through the origin and perpendicular to $5x^2 - 7xy - 3y^2 = 0$ is $3x^2 - 7xy - 5y^2 = 0$. [5]
3. Find the equation of tangents to $3x^2 - 2xy + 7y^2 - 4x = 0$ from the point $(1, -1)$. [5]
4. If PA and PB be the tangents to the conic $\frac{l}{r} = 1 - e\cos\theta$ at angles α and β respectively, then show that PS (S is the focus) bisects the angle ASB. [5]
5. Show that the equation of the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 4 = 0$ at $(4, -2, 2)$ and $(0, 0, -2)$ are parallel. [5]

Group: B

Algebra

Answer any **five** questions from question no. 6-12. [5 x 7 = 35 marks]

6. (a) Let $S = \{x \in \mathbb{C} : x^6 = 1\}$. Does $(S, .)$ form a group, where '.' is the multiplication of complex numbers? [4]

   (b) Let $S = \{(1, 1), (1, -1), (-1, 1), (-1, -1)\}$. Check whether $(S, *)$ is a cyclic group or not, where '$*$' is defined by $(a, b) * (c, d) = (ac, bd)$; $(a, b), (c, d) \in S$. [3]

7. (a) Let $(3\mathbb{Z}, +)$ and $(4\mathbb{Z}, +)$ be two subgroups of the group $(\mathbb{Z}, +)$. Does the union of these two groups form a subgroup of the group $(\mathbb{Z}, +)$? Justify your answer. [2]

   (b) Let $P = \{A \in M_2(\mathbb{R}) : detA \neq 0\}$. Check whether $(P, +, .)$ forms a ring or not, where '+' and '.' are addition of matrices and multiplication of matrices respectively and $M_2(\mathbb{R})$ is the set of all 2×2 real matrices. [3]

   (c) Find the units of the ring $(\mathbb{Z}_8, +, .)$. [2]

8. (a) Examine whether $\{a + b\sqrt{3} : a, b \in \mathbb{R}\}$ forms an integral domain or not. [3]

   (b) Give an example of a finite field containing five elements. [2]

   (c) Prove that the set $S = \{(a, 0) : a \in \mathbb{Z}\}$ is a subring of the ring $\mathbb{Z} \times \mathbb{Z}$. [2]

9. (a) Examine if the set S is a subspace of $\mathbb{R}^3$, where $S = \{(x, y, z) \in \mathbb{R}^3 : x = y = 0\}$. [2]

   (b) Determine k so that the set S is linearly independent in $\mathbb{R}^3$, where $S = \{(1, 2, 1), (k, 3, 1), (2, k, 0)\}$. [3]

   (c) Find the eigenvalues of the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 5 \end{pmatrix}$. [2]

10. (a) Verify Cayley Hamilton Theorem for the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}$. [4]

    (b) If $\{\alpha_1, \alpha_2, \alpha_3\}$ be a basis of a real vector space V and $\beta_1 = \alpha_1 + \alpha_2$, $\beta_2 = 2\alpha_1 + 3\alpha_2 + 4\alpha_3$, $\beta_3 = \alpha_1 + 2\alpha_2 + 3\alpha_3$. Using Replacement theorem, prove that $\{\beta_1, \beta_2, \beta_3\}$ is also a basis of V. [3]

11. (a) Show that $2x^7 - x^4 + 4x^3 - 5 = 0$ has at least four complex roots. [3]

    (b) Solve the equation $x^4 - 8x^3 + 32x^2 - 80x + 100 = 0$, having given that $3 + i$ be a root. [4]

12. (a) If α, β, γ be the roots of $x^3 + px^2 + qx + r = 0$, form the equation whose roots are $\beta + \gamma - 2\alpha$, $\gamma + \alpha - 2\beta$, $\alpha + \beta - 2\gamma$;
    and hence find out the value of $(\beta + \gamma - 2\alpha)(\gamma + \alpha - 2\beta)(\alpha + \beta - 2\gamma)$. [3+1]

(b) If $x = \cos\theta + i\sin\theta$, $y = \cos\theta + i\sin\theta$, and m, n are positive integers, find $(i) x^n + \frac{1}{x^n}$, $(ii) y^m - \frac{1}{y^m}$. [3]

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